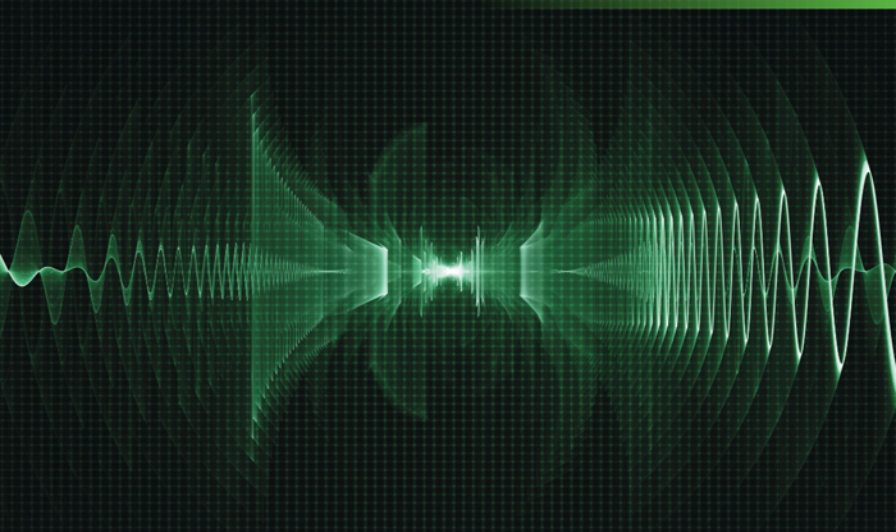


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Sixth Edition in SI Units

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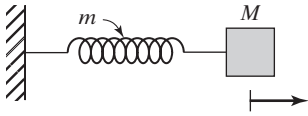


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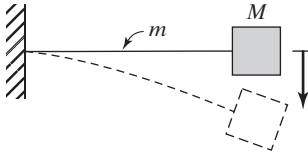
Equivalent Masses, Springs and Dampers

Equivalent masses



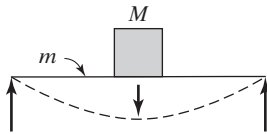
Mass (M) attached at end of spring of mass m

$$m_{eq} = M + \frac{m}{3}$$



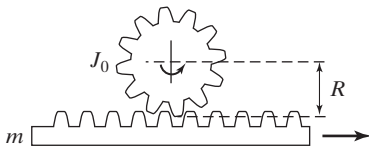
Cantilever beam of mass m carrying an end mass M

$$m_{eq} = M + \frac{33}{140} m$$



Simply supported beam of mass m carrying a mass M at the middle

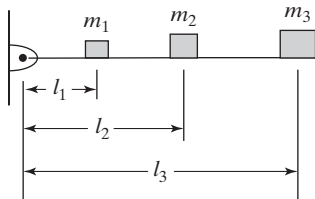
$$m_{eq} = M + 0.5m$$



Coupled translational and rotational masses

$$m_{eq} = m + \frac{J_0}{R^2}$$

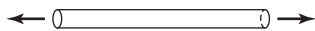
$$J_{eq} = J_0 + mR^2$$



Masses on a hinged bar

$$m_{eq_1} = m_1 + \left(\frac{l_2}{l_1}\right)^2 m_2 + \left(\frac{l_3}{l_1}\right)^2 m_3$$

Equivalent springs



Rod under axial load
(l = length, A = cross sectional area)

$$k_{eq} = \frac{EA}{l}$$



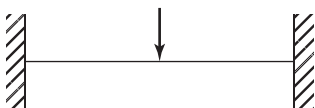
Tapered rod under axial load
(D, d = end diameters)

$$k_{eq} = \frac{\pi EDd}{4l}$$



Helical spring under axial load
(d = wire diameter,
 D = mean coil diameter,
 n = number of active turns)

$$k_{eq} = \frac{Gd^4}{8nD^3}$$



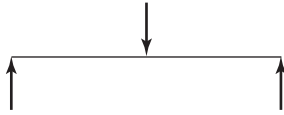
Fixed-fixed beam with load at the middle

$$k_{eq} = \frac{192EI}{l^3}$$



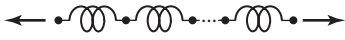
Cantilever beam with end load

$$k_{eq} = \frac{3EI}{l^3}$$



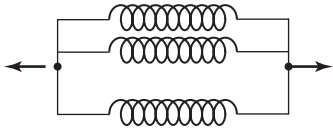
Simply supported beam with load at the middle

$$k_{eq} = \frac{48EI}{l^3}$$



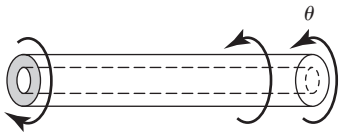
Springs in series

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \dots + \frac{1}{k_n}$$



Springs in parallel

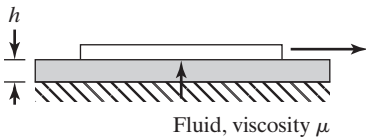
$$k_{eq} = k_1 + k_2 + \dots + k_n$$



Hollow shaft under torsion
(l = length, D = outer diameter,
 d = inner diameter,

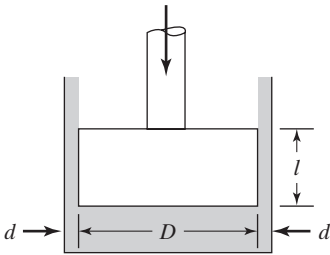
$$k_{eq} = \frac{\pi G}{32l} (D^4 - d^4)$$

Equivalent viscous dampers



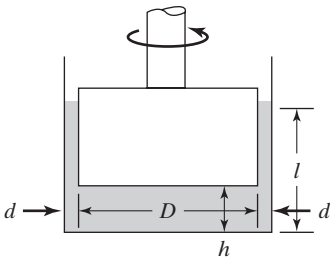
Relative motion between parallel surfaces
(A = area of smaller plate)

$$c_{eq} = \frac{\mu A}{h}$$



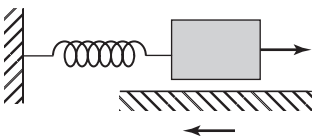
Dashpot (axial motion of a piston in a cylinder)

$$c_{eq} = \mu \frac{3\pi D^3 l}{4d^3} \left(1 + \frac{2d}{D} \right)$$



Torsional damper

$$c_{eq} = \frac{\pi \mu D^2 (l - h)}{2d} + \frac{\pi \mu D^3}{32h}$$



Dry friction (Coulomb damping)
(fN = friction force,
 ω = frequency,
 X = amplitude of vibration)

$$c_{eq} = \frac{4fN}{\pi \omega X}$$

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University of Miami

SI Conversion by

Philip Griffin

University of Limerick, Ireland



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To Lord Sri Venkateswara

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Preface

Changes in This Edition

This book serves as an introduction to the subject of vibration engineering at the undergraduate level. The style of the prior editions has been retained, with the theory, computational aspects, and applications of vibration presented in as simple a manner as possible. As in the previous editions, computer techniques of analysis are emphasized. Expanded explanations of the fundamentals are given, emphasizing physical significance and interpretation that build upon previous experiences in undergraduate mechanics. Numerous examples and problems are used to illustrate principles and concepts. Favorable reactions and encouragement from professors and students have provided me with the impetus to prepare this sixth edition of the book.

Several additions and modifications are made to the fifth edition to make the coverage of vibration more comprehensive and presentation easier to follow in the sixth edition. Most of these additions and modifications were suggested by those who have used the text and by several reviewers. Some of these are indicated below.

- A brief discussion of the anatomy of the human ear and how vibrations are converted into sound by the ear. Experienced engineers can predict the specific cause of malfunction of a machine or engine just by hearing the sound generated by the malfunction.
- Several new applications of vibration are introduced through new problems. The problems are related to the vibration of a child restraint in a child-seat in an automobile, the prediction of injury to head in an automobile accident, the vibratory response of a diver on a high board, the transportation of a precision instrument, and new problems on vibration control.
- The solutions of five examples and eight illustrations are revised for improved presentation so that the reader understands the concept/solution process more easily.
- Additional details are included in the sections related to the response of an undamped system under initial conditions for improved presentation.
- The description and formulation of vibration problems in several different systems of units is considered to obtain the same response of the physical system.
- A new section titled Beams on Elastic Foundation is added along with illustrative examples and problems. This topic finds application in practical situations such as a railway track.
- The stability and vibration of branches of trees with birds sitting on them is considered using the basic principles of mechanics and vibration.
- Nine new examples, 54 new problems (including three new design projects) and 14 new illustrations are added in this edition.

Features of the Book

- Each topic in *Mechanical Vibrations* is self-contained, with all concepts explained fully and the derivations presented with complete details.
- Computational aspects are emphasized throughout the book. MATLAB-based examples as well as several general purpose MATLAB programs with illustrative examples are given in the last section of every chapter. Numerous problems requiring the use of MATLAB or MATLAB programs (given in the text) are included at the end of every chapter.
- Certain topics are presented in a somewhat unconventional manner. The topics of Chapters 9, 10, and 11 fall in this category. Most textbooks discuss isolators, absorbers, and balancing in different chapters. Since one of the main purposes of the study of vibrations is to control vibration response, all topics related to vibration control are given in Chapter 9. The vibration measuring instruments, along with vibration exciters, experimental modal analysis procedure, and machine condition monitoring, are together presented in Chapter 10. Similarly, all the numerical integration methods applicable to single- and multi-degree-of-freedom systems, as well as continuous systems, are unified in Chapter 11.
- Specific features include the following:
 - 252 illustrative examples are given to accompany most topics.
 - 988 review questions are included to help students in reviewing and testing their understanding of the text material. The review questions are in the form of multiple choice questions, questions with brief answers, true-false questions, questions involving matching of related descriptions, and fill-in-the-blank type questions.
 - An extensive set of problems is given in each chapter emphasizing a variety of applications of the material covered in that chapter. In total, there are 1214 problems, with solutions in the instructor's manual.
 - 34 design-project-type problems, many with no unique solution, are given at the end of various chapters.
 - 55 MATLAB programs are included to aid students in the numerical implementation of the methods discussed in the text.
 - Biographical information about 22 scientists and engineers who contributed to the development of the theory of vibrations is presented on the opening pages of all chapters and appendices.
 - MATLAB programs given in the book, answers to problems, and answers to review questions can be found on the Companion Website, www.pearsonglobaleditions.com/rao. The Solutions Manual, with solutions to all problems and hints to design projects, is available to instructors who adopt the text for their courses via download from "Instructor Resources" at www.pearsonglobaleditions.com/rao.

Units and Notation

The SI system of units has been used in the examples and problems. A list of symbols, along with the associated units in the SI system, appears after the Acknowledgments. A brief discussion of SI units as they apply to the field of vibrations is given in Appendix E. Arrows are used over symbols to denote column vectors and square brackets are used to indicate matrices.

Organization of Material

Mechanical Vibrations is organized into 14 chapters and 6 appendices. The reader is assumed to have a basic knowledge of statics, dynamics, strength of materials, and differential equations. Although some background in matrix theory and Laplace transform is desirable, an overview of these topics is given in Appendices C and D, respectively. Chapter 1 starts with a brief discussion of the history and importance of vibrations. The modeling of practical systems for vibration analysis along with the various steps involved in the vibration analysis are discussed. A description of the elementary parts of a vibrating system—stiffness, damping, and mass (inertia)—is given. The basic concepts and terminology used in vibration analysis are introduced. The free vibration analysis of single-degree-of-freedom undamped and viscously damped translational and torsional systems is given in Chapter 2. The graphical representation of characteristic roots and corresponding solutions, the parameter variations, and root locus representations are discussed. Although the root locus method is commonly used in control systems, its use in vibration is illustrated in this chapter. The response under Coulomb and hysteretic damping is also considered. The undamped and damped responses of single-degree-of-freedom systems to harmonic excitations are considered in Chapter 3. The concepts of force and displacement transmissibilities and their application in practical systems are outlined. The transfer function approach, the Laplace transform solution of forced vibration problems, the frequency response, and Bode diagram are presented.

Chapter 4 is concerned with the response of a single-degree-of-freedom system under general forcing function. The roles of Fourier series expansion of a periodic function, convolution integral, Laplace transform, and numerical methods are outlined with illustrative examples. The specification of the response of an underdamped system in terms of peak time, rise time, and settling time is also discussed. The free and forced vibration of two-degree-of-freedom systems is considered in Chapter 5. The self-excited vibration and stability of the system are discussed. The transfer function approach and the Laplace transform solution of undamped and damped systems are also presented with illustrative examples. Chapter 6 presents the vibration analysis of multidegree-of-freedom systems. Matrix methods of analysis are used for the presentation of the theory. The modal analysis procedure is described for the solution of forced vibration problems in this chapter. Several methods of determining the natural frequencies and mode shapes of discrete systems are outlined in Chapter 7. The methods of Dunkerley, Rayleigh, Holzer, Jacobi, and matrix iteration are discussed with numerical examples.

While the equations of motion of discrete systems are in the form of ordinary differential equations, those of continuous or distributed systems are in the form of partial differential equations. The vibration analysis of continuous systems, including strings, bars, shafts, beams, and membranes is given in Chapter 8. The method of separation of variables is presented for the solution of the partial differential equations associated with continuous systems. The Rayleigh and Rayleigh-Ritz methods of finding the approximate natural frequencies are also described with examples. Chapter 9 discusses the various aspects of vibration control, including the problems of elimination, isolation, and absorption. The vibration nomograph and vibration criteria which indicate the acceptable levels of vibration are also presented. The balancing of rotating and reciprocating machines and the whirling of shafts are considered. The active control techniques are also outlined for controlling the response of vibrating systems. The experimental methods used for vibration response measurement are considered in Chapter 10. The hardware used for vibration measurements and signal analysis techniques are described. The machine condition monitoring and diagnosis techniques are also presented.

Chapter 11 presents several numerical integration techniques for finding the dynamic response of discrete and continuous systems. The central difference, Runge-Kutta, Houbolt, Wilson, and Newmark methods are discussed and illustrated. Finite element analysis, with applications involving one-dimensional elements, is discussed in Chapter 12. Bar, rod, and beam elements are used for the static and dynamic analysis of trusses, rods under torsion, and beams. The use of consistent and lumped mass matrices in the vibration analysis is also discussed in

this chapter. Nonlinear vibration problems are governed by nonlinear differential equations and exhibit phenomena that are not predicted or even hinted by the corresponding linearized problems. An introductory treatment of nonlinear vibration, including a discussion of subharmonic and superharmonic oscillations, limit cycles, systems with time-dependent coefficients, and chaos, is given in Chapter 13. The random vibration of linear vibration systems is considered in Chapter 14. The concepts of random process, stationary process, power spectral density, autocorrelation, and wide- and narrow-band processes are explained. The random vibration response of single- and multidegree-of-freedom systems is discussed in this chapter.

Appendices A and B focus on mathematical relationships and deflection of beams and plates, respectively. The basics of matrix theory, Laplace transform, and SI units are presented in Appendices C, D, and E, respectively. Finally, Appendix F provides an introduction to MATLAB programming.

Typical Syllabi

The material of the book provides flexible options for different types of vibration courses. Chapters 1 through 5, Chapter 9, and portions of Chapters 6 constitute a basic course in mechanical vibration. Different emphases/orientations can be given to the course by covering, additionally, different chapters as indicated below:

- Chapter 8 for continuous or distributed systems.
- Chapters 7 and 11 for numerical solutions.
- Chapter 10 for experimental methods and signal analysis.
- Chapter 12 for finite element analysis.
- Chapter 13 for nonlinear analysis.
- Chapter 14 for random vibration.

Alternatively, in Chapters 1 through 14, the text has sufficient material for a one-year sequence of two vibration courses at the senior or dual level.

Expected Course Outcomes

The material presented in the text helps achieve some of the program outcomes specified by ABET (Accreditation Board for Engineering and Technology):

- Ability to apply knowledge of mathematics, science, and engineering:
The subject of vibration, as presented in the book, applies the knowledge of mathematics (differential equations, matrix algebra, vector methods, and complex numbers) and science (statics and dynamics) to solve engineering vibration problems.
- Ability to identify, formulate, and solve engineering problems:
The numerous illustrative examples, problems for practice, and design projects help identify various types of practical vibration problems and develop mathematical models, analyze, solve to find the response, and interpret the results.
- Ability to use the techniques, skills, and modern engineering tools necessary for engineering practice:
 - The application of the modern software, MATLAB, for the solution of vibration problems is illustrated in the last section of each chapter. The basics of MATLAB programming are summarized in Appendix F.

- The use of the modern analysis technique, Finite Element Method, for the solution of vibration problems is covered in a separate chapter (Chapter 12). The finite element method is a popular technique that is used in industry for the modeling, analysis, and solution of complex vibrating systems.
- Ability to design and conduct experiments, as well as to analyze and interpret data:
The experimental methods and analysis of data related to vibration are presented in Chapter 10. The equipment used in conducting vibration experiments, signal analysis, and identification of system parameters from the data are discussed.

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